Nankai University MBA中心

Modern Logistics & Supply Chain Management

ML & SCM

Supply Networks
Minimal Spanning Tree

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Winter 2016

As gold which he cannot spend
will make no man rich,
so knowledge which he cannot apply
will make no man wise.
Samuel Johnson: The Idler No. 84

Learning Objectives

- To know some Graph Theory
  - Terminology network definition, cycles, …
- To understand the minimal spanning tree (MST)
  - To know the solution procedure
  - To state it as a Mathematical Program
  - To be aware of typical MST applications
- To create algorithms

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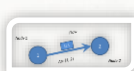
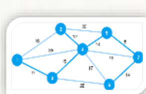
Contents

- Graph Theory
- Minimal Spanning Tree
- Applications





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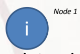
Node

- Node = vertex

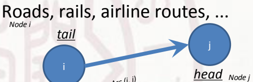


  - Cities, transshipment points, airports, warehouses, ...

Autumn 2016 ML&SCM Graph Theory

Arc
Terminology

- Arc = directed edge
  - Roads, rails, airline routes, ...



  - (*i,j*) *emanates* from *i* and
  - (*i,j*) *terminates* in *j*
  - (*i,j*) is *incident* to *i* and *j*
  - (*i,j*) is an *outgoing* arc of *i*
  - (*i,j*) is an *incoming* arc of j
  - If there is *(i,j)* the node *i* and *j* are *adjacent*

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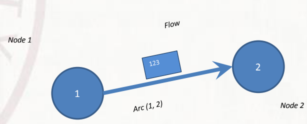
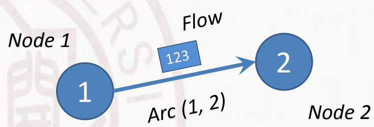
Flow

- Products, freight, intermodal containers, passengers, vehicles, materials, goods ...
- Quantity, weight, cost, distance, time, ...



Autumn 2016 ML&SCM Graph Theory

Graphs & networks



- *directed graph* $G = (N, A)$
  - consists of a set of nodes N and arcs A
- *directed network*
  - Is a directed graph and nodes and/or arcs have associated numerical values
- *subgraph* $G' = (N', A')$
  - if $N' \subseteq N$ and $A' \subseteq A$
- *spannning subgraph*
  - if $N' = N$ and $A' \subseteq A$

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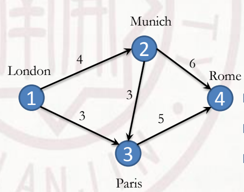
Network example

- Four nodes N, five arcs A and five flows W.
- "London", node 1, termed ***origin or source***
- Arcs identified by beginning and ending node numbers.
- Flow assigned to each arc (distance, time, cost, …)



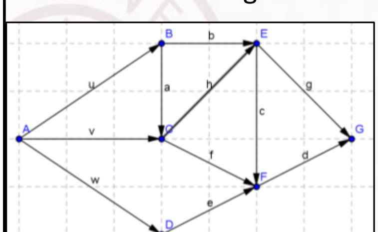
Network of Railroad routes

London, Munich, Paris, Rome

- $N = \{1, 2, 3, 4\}$
- $A = <(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)>$
- $W = <4, 3, 3, 6, 5>$

ML&SCM Graph Theory

Node Degrees



What is the outdegree of A?
3
What is the outdegree of B?
2
What is the indegree of B?
1
What is the degree of B?
3

- *outdegree* = #outgoing arcs
- *indegree* = #incoming arcs
- *degree* = outdegree + indegree

Is the sum of all indgrees the same as the sum of all outdegrees?
yes

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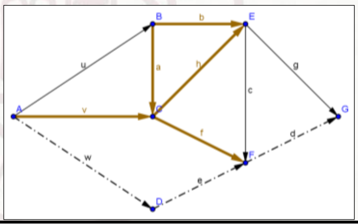
Walks & paths

- A *walk* is a sequence of nodes and arcs n_1-a_1-n_2-a_2-…-n_{r-1}-a_{r-1}-n_r, such that nodes and arcs are a subgraph of (N,A)
- A *path* is a walk without any repetition of nodes.

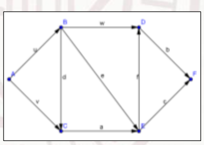


Is A-v-C-h-E-b-a-C-f-F a walk or path?
walk

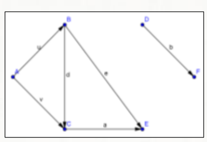
Is A-w-D-e-F-d-G a walk or path?
walk and path

Connected

- Two nodes are *connected* in a graph, if there is at least one path from node *i* to node *j*.
- A *graph* is *connected* if every pair of nodes is connected.



A and F are connected



Graph not connected

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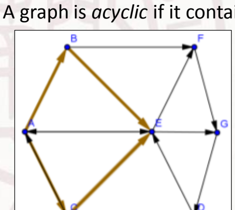
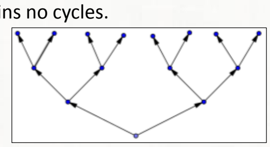
Cycles

- A *cycle* is a closed path of positive length. A cycle is a path $n_1 - n_2$ - … - n_r together with the arc (n_r, n_1) or (n_1, n_r).
- A graph is *acyclic* if it contains no cycles.





Binary tree is acyclic

A-B-E-C-A is a cycle

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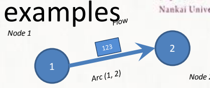
Forests & trees

- A *forest* is an acyclic graph. A graph that contains no cycles is a forest. A forest is a collection of trees.
- A *tree* is an acyclic connect graph. A tree is a connect graph that contains no cycles.

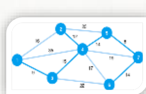
Graph Theory - examples



- Communication Networks
  - Nodes: Telephone exchanges, routers, satellites, computers, ...
  - Arcs: Cables, Fiber optic links, wireless connections
  - Flow: Data, Video, “voice”
- Computer & Electrical circuits
  - Nodes: Processors, gates, registers, intersections, ...
  - Arcs: Wires, resistors, capacitors, inductors, ...
  - Flow: electrical current
- Hydraulic Systems
  - Nodes: Lakes, reservoirs, pumping stations
  - Arcs: pipelines
  - Flow: water, gas, oil, ...

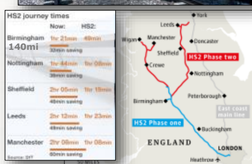
Contents

- Graph Theory
- Minimal Spanning Tree
- Applications



High Speed rail network

- Trains
  - 400m long; 1,100 seats;
  - 250mph (124mph~200km/h)
- Network
  - Distances: several 100 miles
  - Times: <1.5 hours
  - Cost: £42.6bn [June, 2013]
    - £14.4bn contingency



Sources: Guardian, BBC, Gov.uk

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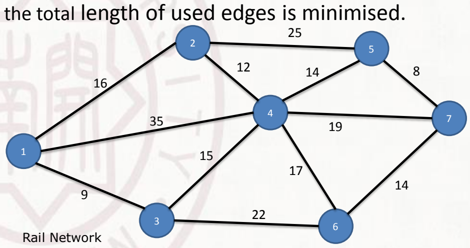
MST Problem Specification

- All nodes have to be connected via edges such that the total length of used edges is minimised.



Rail Network

N = {1, 2, …, 7}
E = <(1,2),(1,3),…,(6,7)>
W = <16, 9, …, 14>

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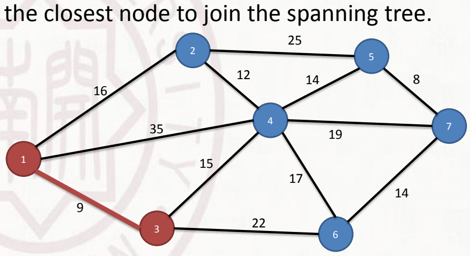
MST – Finding a solution (1/6)

- Start with any node in the network and select the closest node to join the spanning tree.



Spanning Tree with Nodes 1 and 3

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MST – Finding a solution (2/6)

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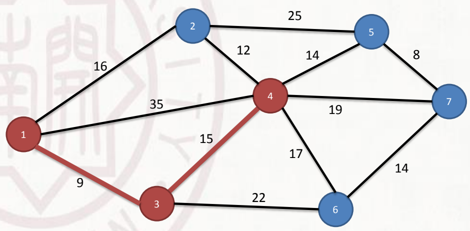
Select the closest node not presently in the spanning tree.



Spanning Tree with Nodes 1, 3, and 4

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MST – Finding a solution (3/6)

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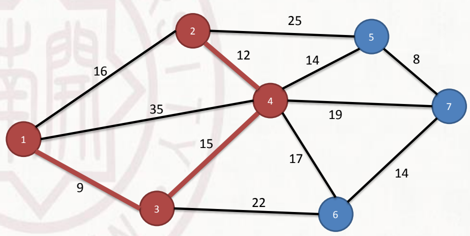
Continue to select the closest node not presently in the spanning tree.



Spanning Tree with Nodes 1, 2, 3, and 4

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MST – Finding a solution (4/6)

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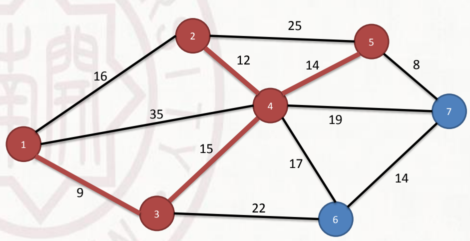
Continue to select the closest node not presently in the spanning tree.



Spanning Tree with Nodes 1, 2, 3, 4, and 5

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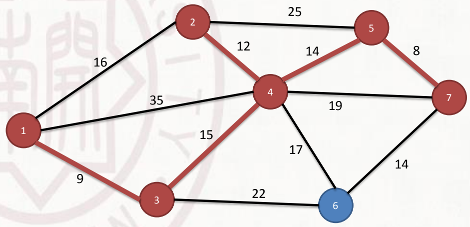
MST – Finding a solution (5/6)

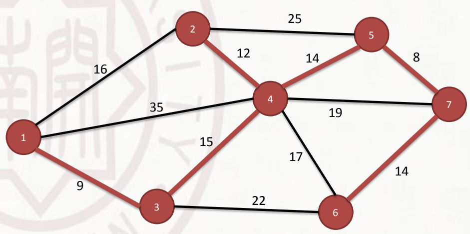
Continue to select the closest node not presently in the spanning area.



Spanning Tree with Nodes 1, 2, 3, 4, 5, and 7

MST – Finding a solution (6/6)

Optimal Solution



Minimal Spanning Tree for Rail network

MST – a solution procedure

1. Select any starting node (usually node 1)
2. Add the node closest to the starting node to form a tree
3. Select the closest node
   - from the remaining nodes that is not in the tree
   - and does not form a cycle
4. Repeat step 3 until all nodes are connected

Advantage: simple to formulate
Disadvantage: Step 3 is inefficient, because distances have to be repeatedly sorted

MST – Kruskal's approach (in words)

1. Sort all edges
2. Go through sorted edge list
   - i.e. take edge from top not used yet
3. Add the edge - if no cycle is formed
4. Stop when all nodes are connected, otherwise continue with 2

Advantage:
- Efficient algorithm,
- "Natural" clustering procedure

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Kruskal: Step 1 sort edges

Arcs

from	to	distance
1	2	16
1	3	9
1	4	35
2	4	12
2	5	25
3	4	15
3	6	22
4	5	14
4	6	17
4	7	19
5	7	8
6	7	14

1. Sort arcs by distances

from	to	distance
5	7	8
1	3	9
2	4	12
4	5	14
6	7	14
3	4	15
1	2	16
4	6	17
4	7	19
3	6	22
2	5	25
1	4	35

Quicksort is one of the fastest and most popular sorting algorithms

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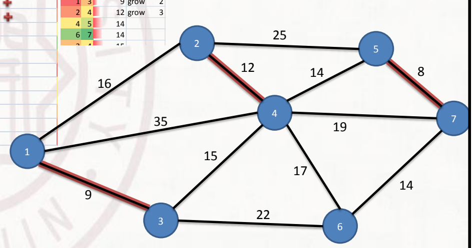
Kruskal – go through edge list

1. Sort arcs by distances

from	to	distance
5	7	8
1	3	9
2	4	12
4	5	14
6	7	14
3	4	15
1	2	16
4	6	17
4	7	19
3	6	22
2	5	25
1	4	35

2. Grow trees

from	to	distance	Step	Tree
5	7	8	grow	1
1	3	9	grow	2
2	4	12	grow	3
4	5	14		
6	7	14		



Shows how the algorithm forms naturally clusters

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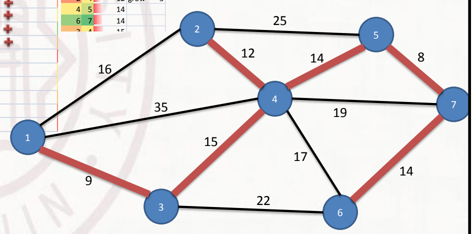
Kruskal – continue iterating through the edges

1. Sort arcs by distances

from	to	distance	
5	7	8	+
1	3	9	+
2	4	12	+
4	5	14	+
6	7	14	+
3	4	15	+
1	2	16	
4	6	17	
4	7	19	
3	6	22	
2	5	25	
1	4	35	

2. Grow trees

from	to	distance	Step	Tree
5	7	8	grow	1
1	3	9	grow	2
2	4	12	grow	3
4	5	14		
6	7	14		



Note that if edge (4,6) would have the distance 14.5, then adding the edge would form the cycle <4, 5, 7, 6, 4>, which we don't want!

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Kruskal's example summary

Arcs

from	to	distance
1	2	16
1	3	9
1	4	35
2	4	12
2	5	25
3	4	15
3	6	22
4	5	14
4	6	17
4	7	19
5	7	8
6	7	14

1. Sort arcs by distances

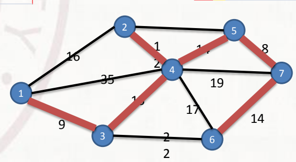
from	to	distance
5	7	8
1	3	9
2	4	12
4	5	14
6	7	14
3	4	15
1	2	16
4	6	17
4	7	19
3	6	22
2	5	25
1	4	35

2. Grow trees

from	to	distance	Step	Tree
5	7	8	grow	1
1	3	9	grow	2
2	4	12	grow	3
4	5	14		
6	7	14		

3. Continue growing trees

from	to	distance	Step	Tree	Nodes
5	7	8	grow tree	1	5,7
1	3	9	grow tree	2	1,3
2	4	12	grow tree	3	2,4
4	5	14	combine	1,3 >> 1	2,4,5,7
6	7	14	grow tree	1	2,4,5,6,7
3	4	15	combine	2,1 >> 1	1,2,3,4,5,6,7
1	2	16			
4	6	17			
4	7	19			
3	6	22			
2	5	25			
1	4	35			



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MST algorithm

- Input: Network (V, E, w)
  – connected graph with m nodes and n edges, $w_i \geq 0$

1. Edge ordering $e_1, e_2, \cdots, e_n$ such that $w(e_1) \leq w(e_2) \leq \cdots \leq w(e_n)$. $E^0 = \emptyset, k = 1$
2. If $H = \left(V, E^{k-1} \cup \{e_k\}\right)$ is acyclic, then $E^k := E^{k-1} \cup \{e_k\}$, otherwise $E^k := E^{k-1}$
3. If $\left|E^k\right| = m - 1$, stop, $\left(V, E^k\right)$ is a spanning tree. Otherwise $k := k + 1$, and return to step 2

Source: Integer and combinatorial optimization - Nemhauser, George L., Wolsey, Laurence A. c1988 (I.3. p60ff)

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MST Algorithm (acyclic)

Kruskal's Algorithm (1956); Dijkstra introduced a solution in 1959

INPUT: *weighted arcs* A
OUTPUT: *minimal spanning tree* T
$A := \text{sort}_w A$ sort acrs by increasing weight
$\forall \quad t \in N : T^t = (T_N^t := t, T_A^t := \emptyset)$ each node is a tree
$i := 0$ arc index
$k := 0$ number of found arcs
while $k < |N| - 1$ not all arcs are found
    $i := i + 1$ increment arc index
    $I := T_N^{\{t\}} \cap_I A_i$ find tree(s) including arc nodes
    $|I| = 2 \Rightarrow$ if two trees then
        $T^{I_1} := T^{I_1} \cup T^{I_2} \cup A_i,$ join the trees
        $T^{I_2} := \emptyset$ destroy not needed tree
        $k := k + 1$ increment number of found arcs

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MST and LP

- x_{ij} … 0-1 variable whether arc (i,j) is selected as part of the MST
- Minimize $\sum_{(i,j)\in A} c_{ij} x_{ij}$
- subject to
  - $\sum_{(i,j)\in A} x_{ij} = n - 1,$
    - cardinality constraint, exactly n-1 arcs
  - $\sum_{(i,j)\in A(S)} x_{ij} = |S| - 1$ for any S of nodes
    - "packing" constraint, ensure no cycles

exponential number of constraints

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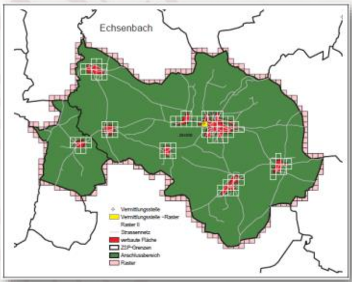
Application – Access Network



Optimal trenching and cabling
- Trenching 80% of costs

Green field approach

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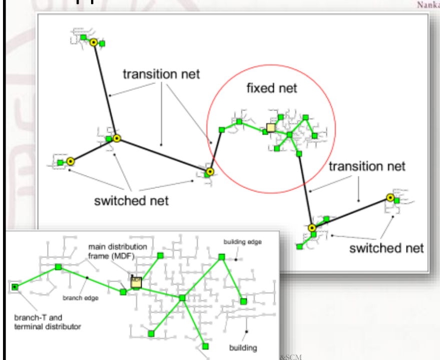
Application – Access Network



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Rail network

- What is the minimal required configuration?

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Recent applications

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Mirror NEWS
UK weather: 220,000 homes without power across England as Storm St Jude lashes country

- What's the most effective way to re-establish the electrical network?

More networks

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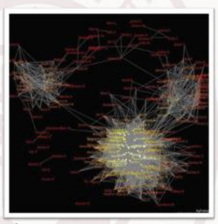
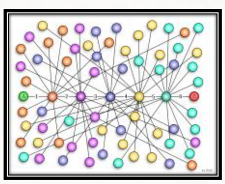

Social networks


Six degrees of separation
Barabási, Albert-László. 2003.
Linked: How Everything is Connected to Everything Else and What It Means for Business, Science, and Everyday Life.

Recap – "the end is near"

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- What is a graph?
  - Nodes and Arcs
  - TV network
- What is a network?
  - A graph and arcs have values ("weights").
  - National grids, telecommunication, social networks
- What is a minimal spanning tree?
  - "The least number of arcs that is necessary to reach all nodes and minimises the total weight."

The End

- Any questions?

References/Literature

- Introduction to management science: a modeling and case studies approach with spreadsheets - Hillier, Frederick S., Hillier, Mark S., Schmedders, Karl, Stephens, Molly
  - Chapter: 6 (p194ff)
- Introduction to operations research - Hillier, Frederick S., Lieberman, Gerald J. 2010
  - Chapter: 9 (p358ff)
- Network flows: theory, algorithms, and applications - Ahuja, Ravindra K., Magnanti, Thomas L., Orlin, James B. c1993
  - Chapter: 13 (p510ff)
- Issues in operations management: [a management science approach] - Taylor, Bernard W., Garn, Wolfgang 2011 (p92ff)
- Integer and combinatorial optimization - Nemhauser, George L., Wolsey, Laurence A. c1988 (I.3. p60ff)

Defintions

- *Nodes* are natural numbers.
- An *arc* is an ordered pair of nodes.
- An *edge* is an unordered pair of nodes.
- A *direct graph* consists of a set of nodes and a set of arcs whose elements are ordered pairs of distinct nodes.
- An *undirect graph* consists of a set of nodes and a set of arcs whose elements are unordered pairs of distinct nodes.
- A *walk* is a sequence of nodes and arcs n_1-a_1-n_2-a_2-…-n_{r-1}-a_{r-1}-n_r, such that nodes and arcs are a subgraph of (N,A)
- A *path* is a walk without any repetition of nodes.
- Two nodes are connected in a graph, if there is at least one path from node i to node j.
- A *graph* is connected if every pair of nodes is connected.
- A *cycle* is a closed path of positive length. A cycle is a path $n_1 – n_2$ - … - n_r together with the arc (n_r, n_1) or (n_1, n_r).
- A graph is *acyclic* if it contains no cycles.
- A *forest* is an acyclic graph. A graph that contains no cycles is a forest. A forest is a collection of trees.
- A *tree* is an acyclic connect graph. A tree is a connect graph that contains no cycles.