Modern Logistics & Supply Chain Management

ML & SCM

Queueing Theory
Part II – M/M/c

*As gold which he cannot spend
will make no man rich,
so knowledge which he cannot apply
will make no man wise.*
Samuel Johnson: The Idler No. 84

I'm British. I know how to queue.
Douglas Adams, The Hitchhiker's Guide to the Galaxy

Dr. Wolfgang Garn
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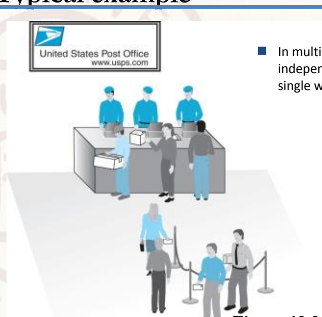
Overview

- Multiple-Server Waiting Line System

Multiple-Server Waiting Line
Typical example

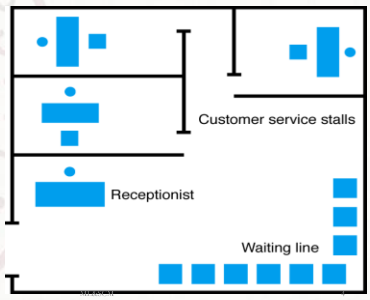


- In multiple-server models, two or more independent servers in parallel serve a single waiting line.

Figure 13.3

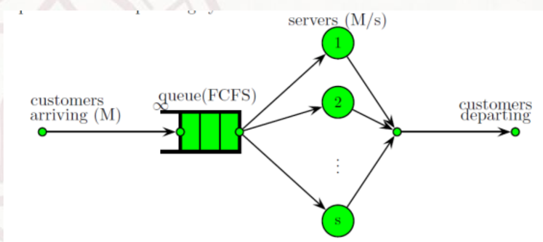
Multiple-Server Waiting Line
Another example

Biggs Department Store service department;
first-come, first-served basis



Customer Service System at Biggs Department Store

Abstraction and Schematic of M/M/c



Multiple-Server Waiting Line
Assumptions and Parameters

- Assumptions for the M/M/c
  - First-come first-served queue discipline
  - Poisson arrivals, exponential service times
  - Infinite calling population.
- Parameter definitions:
  - λ = arrival rate (average number of arrivals per time period)
  - μ = the service rate (average number served per time period) per server (channel)
  - c = number of servers
  - c μ = mean effective service rate for the system (must exceed arrival rate)

Multiple-Server Waiting Line M/M/c - probabilities

$$p_k = \begin{cases} \frac{\lambda^k}{k!\mu^k} p_0 = \frac{\rho^k}{k!} p_0 & \text{if } k < c \\ \frac{\lambda^k}{c!c^{k-c}\mu^k} p_0 = \frac{\rho^k}{c!c^{k-c}} p_0 & \text{otherwise} \end{cases}$$

Probability of *k* customers in the system

$$\sum_{k=0}^{c-1} p_k + \sum_{k=c}^{\infty} p_k = 1$$

Sum of all probabilities must be one

$$p_0 \sum_{k=c}^{\infty} \frac{\lambda^k}{c!c^{k-c}\mu^k} = p_0 \frac{c^c}{c!} \sum_{k=0}^{\infty} \frac{\rho^{k+c}}{c^{k+c}} = p_0 \frac{\rho^c}{c!} \frac{1}{1-\frac{\rho}{c}} = p_0 \frac{\rho^c}{c!} \frac{c}{c-\rho}$$

A few transformations to get p_0

$$p_0 = \left(\sum_{k=0}^{c-1} \frac{\rho^k}{k!} + \frac{c\rho^c}{c!(c-\rho)} \right)^{-1}$$

Probability of no customers in the system

Applications of probabilities

- Operating Characteristic *L*

$$L = \sum_{k=0}^{\infty} k p_k$$

- Probability that a customers will have to wait

$$P[k \geq c] = \sum_{k=c}^{\infty} p_k$$

Erlang C formula

By the way the "Erlang" is also a unit, measures "business" over time, e.g. if three operators are constantly busy over a period of an hour then there is a "traffic" of 3 Erlangs

M/M/c Operating characteristics

Expected number of customers in the system

$$L = \sum_{k=0}^{\infty} k p_k = \frac{\lambda}{\mu} + \frac{\lambda\mu c\rho^c}{c!(c\mu-\lambda)^2} p_0$$

Expected time in system

$$W = \frac{L}{\lambda} = \frac{1}{\mu} + \frac{\mu c\rho^c}{c!(c\mu-\lambda)^2} p_0$$

Expected number of customers in service

$$L_s = \frac{\lambda}{W_s} = \frac{\lambda}{\mu}$$

Expected time in service

$$W_s = \frac{1}{\mu}$$

Expected number of customers in queue

$$L_q = L - L_s = \frac{\lambda\mu c\rho^c}{c!(c\mu-\lambda)^2} p_0$$

Expected time in queue

$$W_q = \frac{L_q}{\lambda} = W - W_s = \frac{\mu c\rho^c}{c!(c\mu-\lambda)^2} p_0$$

Multiple-Server Waiting Line
Biggs Department Store Example (1 of 2)

λ = 10, μ = 4, c = 3

$$P_0 = \frac{1}{\left[\frac{1}{0!}\left(\frac{10}{4}\right)^0 + \frac{1}{1!}\left(\frac{10}{4}\right)^1 + \frac{1}{2!}\left(\frac{10}{4}\right)^2\right] + \frac{1}{3!}\left(\frac{10}{4}\right)^3 \frac{3(4)}{3(4)-10}}$$

= .045 probability of no customers

$$L = \frac{(10)(4)(10/4)^3}{(3-1)![3(4)-10]^2}(.045) + \frac{10}{4}$$

= 6 customers on average in service department

$$W = \frac{6}{10} = 0.60 \text{ hour average customer time in the service department}$$

Winter 2016 ML&SCM 10

Multiple-Server Waiting Line
Biggs Department Store Example (2 of 2)

$$L_q = 6 - \frac{10}{4}$$

= 3.5 customers on the average waiting to be served

$$W_q = \frac{3.5}{10}$$

= 0.35 hour average waiting time in line per customer

$$P_w = \frac{1}{3!}\left(\frac{10}{4}\right)^3 \frac{3(4)}{3(4)-10}(.045)$$

= .703 probability customer must wait for service

Winter 2016 ML&SCM 11

"Quick" Recap

- Which two important systems were analysed?
  - M/M/1 and M/M/c
- Which "law" relates numbers and times?
  - Little's law $L = \lambda W$
- What are the six operating characteristics of a queueing system?
  - Average #customers in system, queue and service
  - Average time spent in system, queue and service

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Qeueing Systems

- Any questions?

Appendix

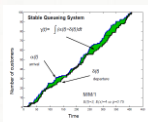
- Production Example
- Stochastic Processes
- Example – Supermarket tills (checkouts)
- Finite queue
- Finite calling problem

Production example (1/2)

- Monitor production line for *time interval T* and keep log of arrival and departure times of each individual item. If T is large the number of *arrivals* would be approximately equal to the *departures N*.
- Arrival rate = total arrivals / total time
  - λ = 100 items per minute = 6,000 items per hour = 240k items / 40 hours

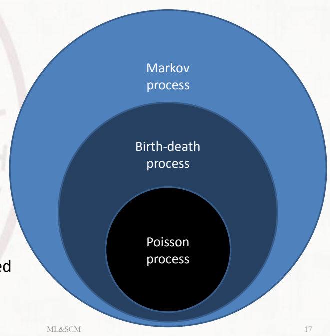
Production example (2/2)

- Mean time spent in production line = “Area” / Number of finished goods = γ x N
  - T = 20 minutes
- Mean number of products in system = arrival rate x mean time spent in system = N/T x γ/N
  - N = λ x T = 100 items/minute x 20 minutes = 2000 items



Stochastic Processes

- Stochastic process
  - Sequence or random variables
- Markovian property
  - Coming events depend only on current state
- Birth death process
  - Transitions to neighbouring states
- Poisson process
  - Independent identical exponentially distributed events



Single-Server Waiting Line System
Effect of Operating Characteristics (1 of 6)

Manager wishes to test several alternatives for reducing customer waiting time:

1. Addition of another employee to pack up purchases
2. Addition of another checkout counter.

Alternative 1: Addition of an employee

(raises service rate from μ = 30 to μ = 40 customers per hour).

- Cost $150 per week, avoids loss of $75 per week for each minute of reduced customer waiting time.
- System operating characteristics with new parameters:

P_o = .40 probability of no customers in the system

L = 1.5 customers on the average in the queuing system

Single-Server Waiting Line System
Effect of Operating Characteristics (2 of 6)

- System operating characteristics with new parameters (continued):

  L_q = 0.90 customer on the average in the waiting line
  W = 0.063 hour average time in the system per customer
  W_q = 0.038 hour average time in the waiting line per customer
  U = .60 probability that server is busy and customer must wait
  I = .40 probability that server is available

Average customer waiting time reduced from 8 to 2.25 minutes worth $431.25 per week.

Single-Server Waiting Line System
Effect of Operating Characteristics (3 of 6)

Alternative 2: Addition of a new checkout counter ($6,000 plus $200 per week for additional cashier).

- λ = 24/2 = 12 customers per hour per checkout counter
- μ = 30 customers per hour at each counter
- System operating characteristics with new parameters:

  P_o = .60 probability of no customers in the system
  L = 0.67 customer in the queuing system
  L_q = 0.27 customer in the waiting line
  W = 0.055 hour per customer in the system
  W_q = 0.022 hour per customer in the waiting line
  U = .40 probability that a customer must wait
  I = .60 probability that server is idle

Single-Server Waiting Line System
Effect of Operating Characteristics (4 of 6)

Savings from reduced waiting time worth:

$500 per week - $200 = $300 net savings per week.

After $6,000 recovered, alternative 2 would provide:

$300 -281.25 = $18.75 more savings per week.

Single-Server Waiting Line System
Effect of Operating Characteristics (5 of 6)

Operating Characteristics	Present System	Alternative I	Alternative II
L	4.00 customers	1.50 customers	0.67 customer
L_q	3.20 customers	0.90 customer	0.27 customer
W	10.00 min	3.75 min	3.33 min
W_q	8.00 min	2.25 min	1.33 min
U	.80	.60	.40

Table 13.1
Operating Characteristics for Each Alternative System

Winter 2016 ML&SCM 22

Single-Server Waiting Line System
Effect of Operating Characteristics (6 of 6)

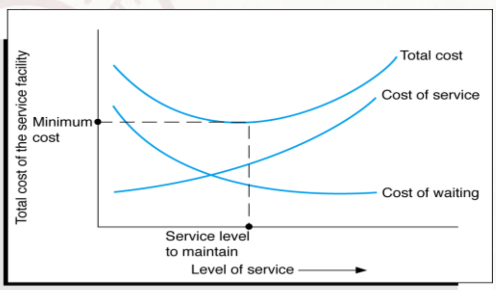


Figure 13.2 Cost Trade-Offs for Service Levels

Winter 2016 ML&SCM 23

Topics

- Elements of Waiting Line Analysis
- The Single-Server Waiting Line System
  - Undefined and Constant Service Times
- Finite Queue Length
- Finite Calling Problem
- The Multiple-Server Waiting Line
- Additional Types of Queuing Systems

Winter 2016 ML&SCM 24

Single-Server Waiting Line System Undefined and Constant Service Times

- *Constant*, rather than exponentially distributed *service times*, occur with machinery and automated equipment.
- Constant service times are a *special case* of the single-server model with undefined service times.
- Queuing formulas:

$$P_0 = 1 - \frac{\lambda}{\mu} \qquad W_q = \frac{L_q}{\lambda}$$

$$L_q = \frac{\lambda^2\sigma^2 + (\lambda/\mu)^2}{2(1-\lambda/\mu)} \qquad W = W_q + \frac{1}{\mu}$$

$$L = L_q + \frac{\lambda}{\mu} \qquad U = \frac{\lambda}{\mu}$$

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Single-Server Waiting Line System Undefined Service Times Example (1 of 2)

- Data: Single fax machine; arrival rate of 20 users per hour, Poisson distributed; undefined service time with mean of 2 minutes, standard deviation of 4 minutes.
- Operating characteristics:

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{20}{30} = .33 \text{ probability that machine not in use}$$

$$L_q = \frac{\lambda^2\sigma^2 + (\lambda/\mu)^2}{2(1-\lambda/\mu)} = \frac{(20)^2(1/15)^2 + (20/30)^2}{2(1-20/30)}$$
$$= 3.33 \text{ employees waiting in line}$$

$$L = L_q + \frac{\lambda}{\mu} = 3.33 + (20/30)$$
$$= 4.0 \text{ employees in line and using the machine}$$

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Single-Server Waiting Line System Undefined Service Times Example (2 of 2)

- Operating characteristics (continued):

$$W_q = \frac{L_q}{\lambda} = \frac{3.33}{20} = 0.1665 \text{ hour} = 10 \text{ minutes waiting time}$$

$$W = W_q + \frac{1}{\mu} = 0.1665 + \frac{1}{30} = 0.1998 \text{ hour}$$
$$= 12 \text{ minutes in the system}$$

$$U = \frac{\lambda}{\mu} = \frac{20}{30} = 67\% \text{ machine utilization}$$

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Single-Server Waiting Line System Constant Service Times Formulas

- In the constant service time model there is no variability in service times; σ = 0.
- Substituting σ = 0 into equations:

$$L_q = \frac{\lambda^2\sigma^2 + (\lambda/\mu)^2}{2(1-\lambda/\mu)} = \frac{\lambda^2 0^2 + (\lambda/\mu)^2}{2(1-\lambda/\mu)} = \frac{(\lambda/\mu)^2}{2(1-\lambda/\mu)} = \frac{\lambda^2}{2\mu(\mu-\lambda)}$$

- All remaining formulas are the same as the single-server formulas.

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Single-Server Waiting Line System Constant Service Times Example

- Car wash servicing one car at a time; constant service time of 4.5 minutes; arrival rate of customers of 10 per hour (Poisson distributed).
- Determine average length of waiting line and average waiting time.

λ = 10 cars per hour, μ = 60/4.5 = 13.3 cars per hour

$$L_q = \frac{\lambda^2}{2\mu(\mu-\lambda)} = \frac{(10)^2}{2(13.3)(13.3-10)} = 1.14 \text{ cars waiting}$$

$$W_q = \frac{L_q}{\lambda} = \frac{1.14}{10} = 0.114 \text{ hour or } 6.84 \text{ minutes waiting time}$$

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Topics

- Elements of Waiting Line Analysis
- The Single-Server Waiting Line System
  - Undefined and Constant Service Times
- Finite Queue Length
- Finite Calling Problem
- The Multiple-Server Waiting Line
- Additional Types of Queuing Systems

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Finite Queue Length

- In a finite queue, the *length of the queue is limited*.
- Operating characteristics, where M is the maximum number in the system:

$$P_0 = \frac{1-\lambda/\mu}{1-(\lambda/\mu)^{M+1}} \qquad P_n = (P_0)\left(\frac{\lambda}{\mu}\right)^n \text{ for } n \le M$$

$$L = \frac{\lambda/\mu}{1-\lambda/\mu} - \frac{(M+1)(\lambda/\mu)^{M+1}}{1-(\lambda/\mu)^{M+1}} \qquad L_q = L - \frac{\lambda(1-P_M)}{\mu}$$

$$W = \frac{L}{\lambda(1-P_M)} \qquad W_q = W - \frac{1}{\mu}$$

Winter 2016 ML&SCM 31

Finite Queue Length Example (1 of 2)

Metro Quick Lube single bay service; space for one vehicle in service and three waiting for service; mean time between arrivals of customers is 3 minutes; mean service time is 2 minutes; both inter-arrival times and service times are exponentially distributed; maximum number of vehicles in the system equals 4.

Operating characteristics for λ = 20, μ = 30, M = 4:

$$P_0 = \frac{1-\lambda/\mu}{1-(\lambda/\mu)^{M+1}} = \frac{1-20/30}{1-(20/30)^5} = .38 \text{ probability that system is empty}$$

$$P_M = (P_0)\left(\frac{\lambda}{\mu}\right)^{n=M} = (.38)\left(\frac{20}{30}\right)^4 = .076 \text{ probability that system is full}$$

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Finite Queue Length Example (2 of 2)

Average queue lengths and waiting times:

$$L = \frac{\lambda/\mu}{1-\lambda/\mu} - \frac{(M+1)(\lambda/\mu)^{M+1}}{1-(\lambda/\mu)^{M+1}}$$

$$L = \frac{20/30}{1-20/30} - \frac{(5)(20/30)^5}{1-(20/30)^5} = 1.24 \text{ cars in the system}$$

$$L_q = L - \frac{\lambda(1-P_M)}{\mu} = 1.24 - \frac{20(1-.076)}{30} = 0.62 \text{ cars waiting}$$

$$W = \frac{L}{\lambda(1-P_M)} = \frac{1.24}{20(1-.076)} = 0.067 \text{ hours waiting in the system}$$

$$W_q = W - \frac{1}{\mu} = 0.067 - \frac{1}{30} = 0.033 \text{ hour waiting in line}$$

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Finite Calling Population

- In a finite calling population there is a limited number of potential customers that can call on the system.
- Operating characteristics for system with Poisson arrival and exponential service times:

$$P_0 = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$

where N = population size, and n = 1, 2,...N

$$P_n = \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0 \qquad L_q = N - \left(\frac{\lambda-\mu}{\lambda}\right)(1-P_0)$$

$$L = L_q + (1-P_0) \qquad W_q = \frac{L_q}{(N-L)\lambda} \qquad W = W_q + \frac{1}{\mu}$$

Winter 2016 ML&SCM 34

Finite Calling Population Example (1 of 2)

Wheelco Manufacturing Company; 20 machines; each machine operates an average of 200 hours before breaking down; average time to repair is 3.6 hours; breakdown rate is Poisson distributed, service time is exponentially distributed.

Is repair staff sufficient?

λ = 1/200 hour = .005 per hour

μ = 1/3.6 hour = .2778 per hour

N = 20 machines

Winter 2016 ML&SCM 35

Finite Calling Population Example (2 of 2)

$$P_0 = \frac{1}{\sum_{n=0}^{20} \frac{20!}{(20-n)!}\left(\frac{.005}{.2778}\right)^n} = .652$$

$$L_q = 20 - \frac{.005+.2778}{.005}(1-.652) = .169 \text{ machines waiting}$$

$$L = .169 + (1-.652) = .520 \text{ machines in the system}$$

$$W_q = \frac{.169}{(20-.520)(.005)} = 1.74 \text{ hours waiting for repair}$$

$$W = 1.74 + \frac{1}{.2778} = 5.33 \text{ hours in the system}$$

…System seems woefully inadequate.

Winter 2016 ML&SCM 36

Topics

- Elements of Waiting Line Analysis
- The Single-Server Waiting Line System
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Additional Types of Queuing Systems (1 of 2)

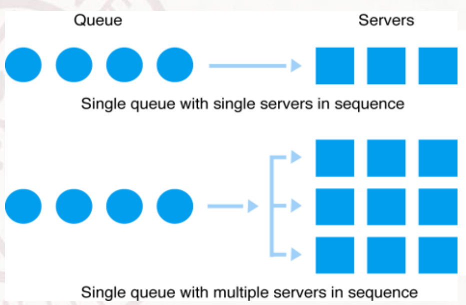

Figure 13.4 Single Queues with Single and Multiple Servers in Sequence

Additional Types of Queuing Systems (2 of 2)

Other items contributing to queuing systems:

- Systems in which customers *balk* from entering system, or leave the line (*renege*).
- Servers who provide service in other than first-come, first-served manner
- Service times that are not exponentially distributed or are undefined or constant
- Arrival rates that are not Poisson distributed
- *Jockeying* (i.e., moving between queues)

Example Problem Solution (1 of 5)

Problem Statement: Citizens Northern Savings Bank loan officer customer interviews.

Customer arrival rate of four per hour, Poisson distributed; officer interview service time of 12 minutes per customer.

1. Determine operating characteristics for this system.
2. Additional officer creating a multiple-server queuing system with two channels. Determine operating characteristics for this system.

Example Problem Solution (2 of 5)

Solution:

Step 1: Determine Operating Characteristics for the Single-Server System

λ = 4 customers per hour arrive, μ = 5 customers per hour are served

$P_o = (1 - \lambda / \mu) = (1 - 4 / 5) = .20$ probability of no customers in the system

$L = \lambda / (\mu - \lambda) = 4 / (5 - 4) = 4$ customers on average in the queuing system

$L_q = \lambda^2 / \mu(\mu - \lambda) = 4^2 / 5(5 - 4) = 3.2$ customers on average in the waiting line

Example Problem Solution (3 of 5)

Step 1 (continued):

$W = 1 / (\mu - \lambda) = 1 / (5 - 4) = 1$ hour on average in the system

$W_q = \lambda / \mu(u - \lambda) = 4 / 5(5 - 4) = 0.80$ hour (48 minutes) average time in the waiting line

$P_w = \lambda / \mu = 4 / 5 = .80$ probability the new accounts officer is busy and a customer must wait

Example Problem Solution (4 of 5)

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Step 2: Determine the Operating Characteristics for the Multiple-Server System.

λ = 4 customers per hour arrive; μ = 5 customers per hour served; c = 2 servers

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)}$$

= .429 probability no customers in system

$$L = \frac{\lambda\mu(\lambda/\mu)^c}{(c-1)!(c\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$$

= 0.952 average number of customers in the system

Winter 2016 ML&SCM 43

Example Problem Solution (5 of 5)

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Step 2 (continued):

$$L_q = L - \frac{\lambda}{\mu}$$

= 0.152 average number of customers in the queue

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

= 0.038 hour average time customer is in the queue

$$P_w = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu-\lambda} Po$$

= .229 probability customer must wait for service

Winter 2016 ML&SCM 44

Box 31.2 M/M/m Queue

1. Parameters:
   λ = arrival rate in jobs per unit time
   μ = service rate in jobs per unit time
   m = number of servers
2. Traffic intensity: $\rho = \lambda/(m\mu)$
3. The system is stable if the traffic intensity ρ is less than 1.
4. Probability of zero jobs in the system:

$$p_0 = \left[1 + \frac{(m\rho)^m}{m!(1-\rho)} + \sum_{n=1}^{m-1}\frac{(m\rho)^n}{n!}\right]^{-1}$$

5. Probability of n jobs in the system:

$$p_n = \begin{cases} p_0 \frac{(m\rho)^n}{n!}, & n < m \\ p_0 \frac{\rho^n m^m}{m!}, & n \geq m \end{cases}$$

6. Probability of queueing:

$$\varrho = P(\geq m \text{ jobs}) = \frac{(m\rho)^m}{m!(1-\rho)}p_0$$

   In the remaining formulas below we will use ϱ as defined here.
7. Mean number of jobs in the system: $E[n] = m\rho + \rho\varrho/(1-\rho)$
8. Variance of number of jobs in the system:

$$\text{Var}[n] = m\rho + \rho\varrho\left[\frac{1+\rho-\rho\varrho}{(1-\rho)^2} + m\right]$$

9. Mean number of jobs in the queue: $E[n_q] = \rho\varrho/(1-\rho)$
10. Variance of number of jobs in the queue: $\text{Var}[n_q] = \varrho\rho(1+\rho-\varrho\rho)/(1-\rho)^2$
11. Average utilization of each server: $U = \lambda/(m\mu) = \rho$
12. Cumulative distribution function of response time:

$$F(r) = \begin{cases} 1 - e^{-\mu r} - \frac{\varrho}{1-m+m\rho}e^{-m\mu(1-\rho)r} - e^{-\mu r}, & \rho \neq (m-1)/m \\ 1 - e^{-\mu r} - \varrho\mu r e^{-\mu r}, & \rho = (m-1)/m \end{cases} \quad r > 0$$

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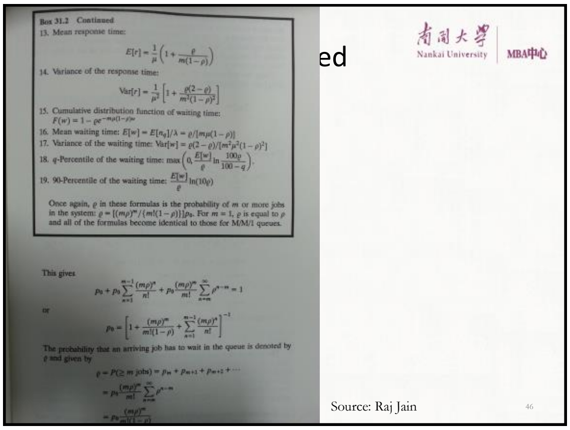
Source: Raj Jain

45



Box 31.2 Continued

13. Mean response time:

$$E[r] = \frac{1}{\mu}\left(1 + \frac{\varrho}{m(1-\rho)}\right)$$

14. Variance of the response time:

$$\mathrm{Var}[r] = \frac{1}{\mu^2}\left[1 + \frac{\varrho(2-\varrho)}{m^2(1-\rho)^2}\right]$$

15. Cumulative distribution function of waiting time: $F(w) = 1 - \varrho e^{-m\mu(1-\rho)w}$
16. Mean waiting time: $E[w] = E[n_q]/\lambda = \varrho/[m\mu(1-\rho)]$
17. Variance of the waiting time: $\mathrm{Var}[w] = \varrho(2-\varrho)/[m^2\mu^2(1-\rho)^2]$
18. q-Percentile of the waiting time: $\max\left(0, \frac{E[w]}{\varrho}\ln\frac{100\varrho}{100-q}\right)$.
19. 90-Percentile of the waiting time: $\frac{E[w]}{\varrho}\ln(10\varrho)$

Once again, ϱ in these formulas is the probability of m or more jobs in the system: $\varrho = [(m\rho)^m/\{m!(1-\rho)\}]p_0$. For $m = 1$, ϱ is equal to ρ and all of the formulas become identical to those for M/M/1 queues.

This gives

$$p_0 + p_0\sum_{n=1}^{m-1}\frac{(m\rho)^n}{n!} + p_0\frac{(m\rho)^m}{m!}\sum_{n=m}^{\infty}\rho^{n-m} = 1$$

or

$$p_0 = \left[1 + \frac{(m\rho)^m}{m!(1-\rho)} + \sum_{n=1}^{m-1}\frac{(m\rho)^n}{n!}\right]^{-1}$$

The probability that an arriving job has to wait in the queue is denoted by ϱ and given by

$$\varrho = P(\geq m \text{ jobs}) = p_m + p_{m+1} + p_{m+2} + \cdots$$
$$= p_0\frac{(m\rho)^m}{m!}\sum_{n=m}^{\infty}\rho^{n-m}$$
$$= p_0\frac{(m\rho)^m}{m!(1-\rho)}$$