Modern Logistics & Supply Chain Management

ML & SCM

Inventory Systems
Part II

Dr. Wolfgang Garn
Winter, 2016

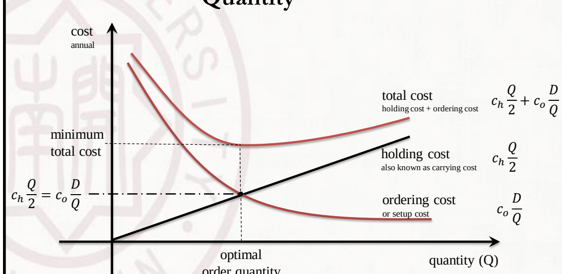
As gold which he cannot spend
will make no man rich,
so knowledge which he cannot apply
will make no man wise.
Samuel Johnson: The Idler No. 84

Learning Objectives

- To use EOQ for non-instantaneous receipt
  - and apply to production scenarios
- To have an idea about EOQ and shortage
- To understand the reorder point
- To apply reorder point model

Winter 2016 ML&SCM 2

Basic EOQ Model
EOQ = Economic Order Quantity



$$Q = \sqrt{2D\frac{c_0}{c_c}}$$

Winter 2016 ML&SCM 3

EOQ with non-instantaneous receipt

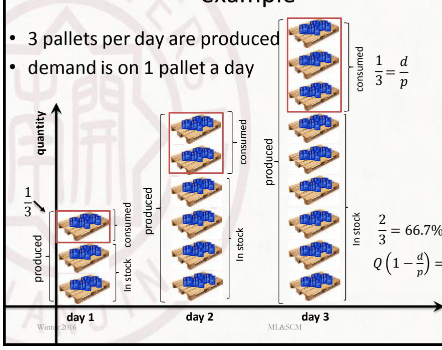
- Orders are received gradually
  - e.g. due to production capacity limits
- EOQ with non-instantaneous receipt

  = production lot size model

  = gradual usage model

Production lot size model example

- 3 pallets per day are produced
- demand is on 1 pallet a day



$\frac{1}{3} = \frac{d}{p}$

Order quantity = Q
= production lot size
= 9 pallets

production rate = p
= 3 pallets/day

Demand rate = d
= 1 pallet/day

$\frac{2}{3} = 66.7\% = 1 - \frac{d}{p}$

$Q\left(1 - \frac{d}{p}\right) = 9\frac{2}{3} = 6$ pallets

EOQ Model
Non-Instantaneous Receipt (2 of 2)

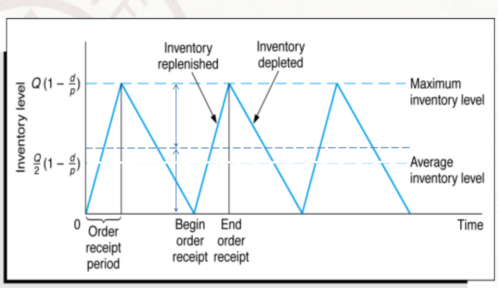
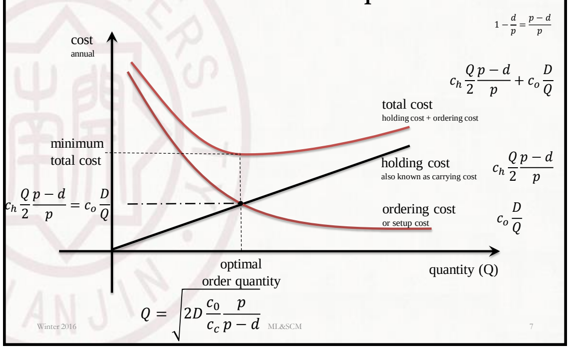


Figure 16.6 The EOQ Model with Non-Instantaneous Order Receipt

EOQ Model with non-instantaneous receipt



Winter 2016 ML&SCM 7

Non-Instantaneous Receipt Model
Model Formulation (1 of 2)

p =daily rate at which the order is received over time
d = daily rate at which inventory is demanded

Maximum inventory level $= Q\left(1-\frac{d}{p}\right)$

Average inventory level $= \frac{Q}{2}\left(1-\frac{d}{p}\right)$

Total carrying cost $= C_c\frac{Q}{2}\left(1-\frac{d}{p}\right)$

Total annual inventory cost$=C_o\frac{D}{Q}+C_c\frac{Q}{2}\left(1-\frac{d}{p}\right)$

Winter 2016 ML&SCM 8

Non-Instantaneous Receipt Model
Model Formulation (2 of 2)

$C_c\frac{Q}{2}\left(1-\frac{d}{p}\right)=C_o\frac{D}{Q}$ **at lowest point of total cost curve**

Optimal order size: $Q_{opt}=\sqrt{\frac{2C_oD}{C_c(1-d/p)}}$

Winter 2016 ML&SCM 9

Production lot size model

- A company processes rice. The setup costs for pure Basmati rice is $c_o = £150$. This includes the cleaning. The annual holding costs are $c_h = £0.75$ per bag. The company operates 311 days a year; and has a yearly demand of 10,000 bags. The factories capacity is 150 bags per day.

What is the optimal order (production) size?

Demand rate: $d = \frac{D}{t} = \frac{10{,}000}{311} = 32.2$ bags per day

Production rate: $p = 150$ bags per day

Optimal production size:

$$Q_{opt} = \sqrt{2D\frac{c_o}{c_h}\frac{p-d}{p}} = \sqrt{2\ 10{,}000\frac{150}{0.75}\frac{150-32.2}{150}} = 2{,}257 \text{ bags}$$

Winter 2016 ML&SCM 10

Production lot size model

continued

- Total cost $= c_h\frac{Q}{2}\frac{p-d}{p} + c_o\frac{D}{Q} = 0.75\frac{10{,}000}{2}\frac{150-32.2}{150} + 150\frac{10{,}000}{2{,}257} = £1{,}329$
  - minimises annual inventory cost
- How long does it take to produce 2,257 bags?
  - Knowing the production rate is 150 bags per day
  - $\frac{Q}{p} = \frac{2{,}257}{150} = 15.1$ days
- How many production runs (orders) are there in a year?
  - Knowing the demand is 10,000 bags per year
  - $\frac{D}{Q} = \frac{10{,}000}{2{,}257} = 4.43$ runs
- What is the maximum inventory level?
  - $Q\left(1-\frac{d}{p}\right) = 2{,}257\left(1-\frac{32.2}{150}\right) = 1{,}772$ bags

Winter 2016 ML&SCM 11

EOQ Model with Shortages Description (1 of 2)

- In the EOQ model with shortages, the assumption that shortages cannot exist is relaxed.
- Assumed that unmet *demand can be backordered* with all demand eventually satisfied.

Winter 2016 ML&SCM 12

EOQ Model with Shortages
Description (2 of 2)

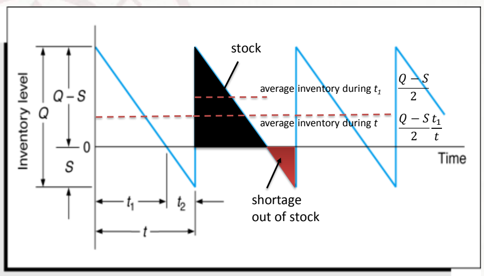


Figure 16.7 The EOQ Model with Shortages

EOQ Model with Shortages
Model Formulation (1 of 2)

Total shortage costs $= C_s \frac{S^2}{2Q}$ Total carrying costs $= C_c \frac{(Q-S)^2}{2Q}$

Total ordering cost $= C_o \frac{D}{Q}$

Total inventory cost $= C_s \frac{S^2}{2Q} + C_c \frac{(Q-S)^2}{2Q} + C_o \frac{D}{Q}$

Optimal order quantity $= Q_{opt} = \sqrt{\frac{2C_o D}{C_c}\left(\frac{C_s + C_c}{C_s}\right)}$

Shortage level $= S_{opt} = Q_{opt}\left(\frac{C_c}{C_c + C_s}\right)$

EOQ Model with Shortages
Model Formulation (2 of 2)

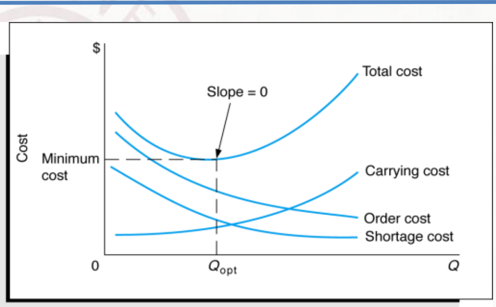


Figure 16.8 Cost Model with Shortages

Reorder Point Model

- Purpose
  - Determines the time to order when the stock level drops below a certain threshold
- Factors
  - Possible variability in delivery lead time
  - Possible variability in demand

Reorder Point (1 of 4)

- The ***reorder point*** is the inventory level at which a new order is placed.
- Order must be made while there is enough stock in place to cover demand during lead time.
- Formulation:

  R = dL

  where d = demand rate per time period

  L = lead time
- For the pure Basmati rice production example:
  - with lead time 10 days

  R = dL = (10,000/311)(10) = 321.5 bags

Reorder Point (2 of 4)

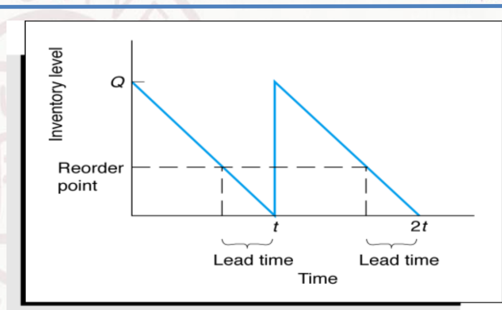


Figure 16.9 Reorder Point and Lead Time

Reorder Point (3 of 4)

南开大学 Nankai University MBA中心

- Inventory level might be depleted at slower or faster rate during lead time.
- When demand is uncertain, ***safety stock*** is added as a hedge against stockout.

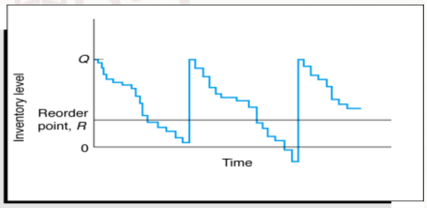


Figure 16.10 Inventory Model with Uncertain Demand

Winter 2016 ML&SCM 19

Reorder Point (4 of 4)

南开大学 Nankai University MBA中心

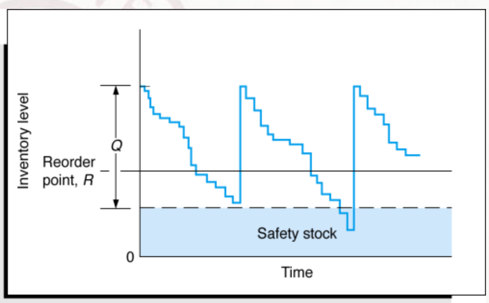


Figure 16.11 Inventory model with safety stock

Winter 2016 ML&SCM 20

The reorder point method of stock control

南开大学 Nankai University MBA中心

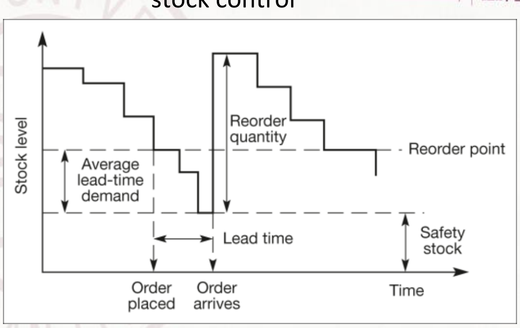


Figure 5.6

Christopher, Logistics and supply chain management, 4th edition © Pearson Education Ltd

Determining Safety Stocks Using Service Levels

- ***Service level*** is a probability that ensures inventory on hand is sufficient to meet demand during lead time (probability stockout will not occur).
- The higher the probability inventory will be on hand, the more likely customer demand will be met.
- Service level of 90% means there is a .90 probability that demand will be met during lead time and .10 probability of a stockout.

Four classic ROP Models

Const. d + const. L	var. d + const. L
$R = d\,L$	$R = \bar{d}L + Z\sigma_d\sqrt{L}$
Const. d + var. L	**var. d + var. L**
$R = d\bar{L} + dZ\sigma_L$	$R = \bar{d}\bar{L} + Z\sqrt{\sigma_d{}^2\bar{L} + \sigma_L{}^2\bar{d}^2}$

d ... daily demand
L ... Lead time

Reorder Point with Variable Demand (1 of 2)

$R = \bar{d}L + Z\sigma_d\sqrt{L}$

where:

R = reorder point

$\bar{d}$ = average daily demand

L = lead time

σ_d = the standard deviation of daily demand

Z = number of standard deviations corresponding to service level probability

$Z\sigma_d\sqrt{L}$ = safety stock

Reorder Point with Variable Demand (2 of 2)

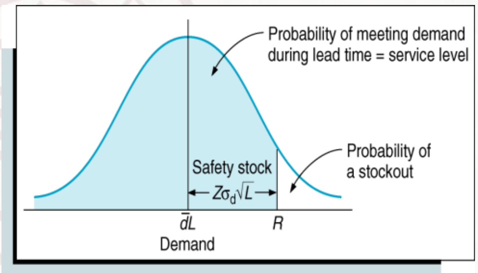


Figure 16.12 Reorder Point for a Service Level

Reorder Point with Variable Demand Example

I-75 Carpet Discount Store Super Shag carpet.

For following data, determine reorder point and safety stock for service level of 95%.

$\bar{d} = 30$ yd per day
L = 10 days
σ_d = 5 yd per day

For 95% service level, Z = 1.65 (Table A-1, appendix A)

$$R = \bar{d}L + Z\sigma_d\sqrt{L} = 30(10) + (1.65)(5)(\sqrt{10}) = 300 + 26.1$$
$$= 326.1 \text{ yd}$$

Safety stock is second term in reorder point formula : 26.1.

Reorder Point with Variable Lead Time

For constant demand and variable lead time:

$$R = d\bar{L} + Zd\sigma_L$$

where:

d = constant daily demand
$\bar{L}$ = average lead time
σ_L = standard deviation of lead time
$d\sigma_L$ = standard deviation of demand during lead time
$Zd\sigma_L$ = safety stock

Reorder Point with Variable Lead Time Example

Carpet Discount Store:

$d = 30$ yd per day

$\bar{L} = 10$ days

$\sigma_L = 3$ days

$Z = 1.65$ for a 95% service level

$$R = d\bar{L} + Zd\sigma_L$$
$$= (30)(10) + (1.65)(30)(3)$$
$$= 300 + 148.5$$
$$= 448.5 \text{ yd}$$

Winter 2016 ML&SCM 28

Reorder Point Variable Demand and Lead Time

When both demand and lead time are variable:

$$R = \bar{d}\bar{L} + Z\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$$

where: $\bar{d}$ = average daily demand
$\bar{L}$ = average lead time

$\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$ = standard deviation of demand during lead time

$Z\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$ = safety stock

Winter 2016 ML&SCM 29

Reorder Point Variable Demand and Lead Time Example

Carpet Discount Store:

$\bar{d} = 30$ yd per day
$\sigma_d = 5$ yd per day
$\bar{L} = 10$ days
$\sigma_L = 3$ days
$Z = 1.65$ for 95% service level

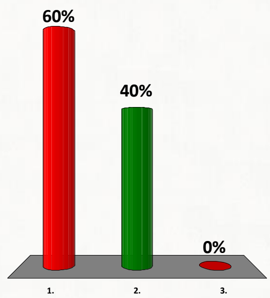
$$R = \bar{d}\bar{L} + Z\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$$

$$= (30)(10) + (1.65)\sqrt{(5)(5)(10) + (3)(3)(30)(30)}$$
$$= 300 + 150.8$$
$$= 450.8 \text{ yds}$$

Winter 2016 ML&SCM 30

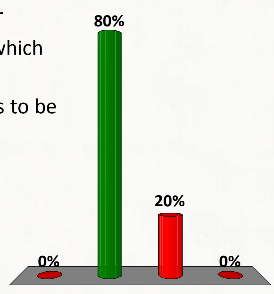
What is a production lot size model?

1. The Basic EOQ
2. An EOQ with non-instantaneous receipt
3. Reorder point model



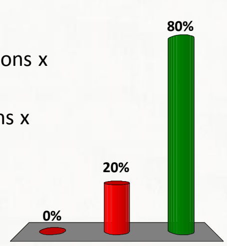
What is the reorder point?

1. The time when to order
2. The inventory level at which an order is initiated
3. The quantity that needs to be ordered
4. The required lead time



How do you compute the safety stock in a Reorder Point model with uncertain demand?

1. Average demand x lead time
2. Z Lead time Standard Deviations x demand
3. Z demand standard deviations x square root of lead time



Assume underlying Normal distribution

Recap – "the end is near"

- What do inventory models minimise?
  - Total costs
- The production lot size model allows…
  - Gradual production
- What is the reorder point?
  - The quantity level at which an order is placed.
  - It depends on the demand and lead time.

Winter 2016 ML&SCM 34

The End

- Any questions?

Winter 2016 ML&SCM 35

Appendix

- EOQ with shortage
- EOQ with discounts
- Periodic Inventory Systems
- Reorder Point models
- ROP with Excel

Winter 2016 ML&SCM 36

EOQ Model with Shortages
Model Formulation (1 of 3)

I-75 Carpet Discount Store allows shortages; shortage cost C_s is $2/yard per year.

C_o = $150
C_c = $0.75 per yd
C_s = $2 per yd
D = 10,000 yd

Optimal order quantity:

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}\left(\frac{C_s + C_c}{C_s}\right)} = \sqrt{\frac{2(150)(10{,}000)}{0.75}\left(\frac{2+0.75}{2}\right)} = 2{,}345.2 \text{ yd}$$

Winter 2016 ML&SCM 37

EOQ Model with Shortages
Model Formulation (2 of 3)

Shortage level:

$$S_{opt} = Q_{opt}\left(\frac{C_c}{C_c + C_s}\right) = 2{,}345.2\left(\frac{0.75}{2+0.75}\right) = 639.6 \text{ yd}$$

Total inventory cost:

$$TC = C_s\frac{S^2}{2Q} + C_c\frac{(Q-S)^2}{2Q} + C_o\frac{D}{Q}$$

$$= \frac{(2)(639.6)^2}{2(2{,}345.2)} + \frac{(0.75)(1{,}705.6)^2}{2(2{,}345.2)} + \frac{(150)(10{,}000)}{2{,}345.2}$$

= $174.44 + 465.16 + 639.60 = $1,279.20

Winter 2016 ML&SCM 38

EOQ Model with Shortages
Model Formulation (3 of 3)

Number of orders $= \frac{D}{Q} = \frac{10{,}000}{2{,}345.2} = 4.26$ **orders per year**

Maximum inventory level $= Q - S = 2{,}345.2 - 639.6 = 1{,}705.6$ **yd**

Time between orders $= t = \frac{\text{days per year}}{\text{number of orders}} = \frac{311}{4.26} = 73.0$ **days**

Time during which inventory is on hand

$= t_1 = \frac{Q-S}{D} = \frac{2{,}345.2\text{-}639.6}{10{,}000} = 0.171$ **or 53.2 days**

Time during which there is a shortage

$= t_2 = \frac{S}{D} = \frac{639.6}{10{,}000} = 0.064$ **year or 19.9 days**

Winter 2016 ML&SCM 39

Quantity Discounts

- Price discounts are often offered if a predetermined number of units is ordered or when ordering materials in high volume.
- Basic EOQ model used with purchase price added:

$$TC = C_o \frac{D}{Q} + C_c \frac{Q}{2} + PD$$

where: P = per unit price of the item
D = annual demand

- Quantity discounts are evaluated under two different scenarios:
  - With constant carrying costs
  - With carrying costs as a percentage of purchase price

Winter 2016 ML&SCM 40

Quantity Discounts with Constant Carrying Costs - Analysis Approach

- Optimal order size is the same regardless of the discount price.
- The total cost with the optimal order size must be compared with any lower total cost with a discount price to determine which is the lesser.

Winter 2016 ML&SCM 41

Quantity Discounts with Constant Carrying Costs - Example (1 of 2)

University bookstore: For following discount schedule offered by Comptek, should bookstore buy at the discount terms or order the basic EOQ order size?

Quantity	Price
1- 49	$1,400
50 – 89	1,100
90 +	900

Determine optimal order size and total cost:

C_o = $2,500 C_c = $190 per unit D = 200

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2(2{,}500)(200)}{190}} = 72.5$$

Winter 2016 ML&SCM 42

Quantity Discounts with Constant Carrying Costs - Example (2 of 2)

- Compute total cost at eligible discount price ($1,100):

$$TC_{\min} = \frac{C_o D}{Q_{opt}} + C_c \frac{Q_{opt}}{2} + PD$$

$$= \frac{(2{,}500)(200)}{(72.5)} + (190)\frac{(72.5)}{2} + (1{,}100)(200) = \$233{,}784$$

- Compare with total cost of with order size of 90 and price of $900:

$$TC = \frac{C_o D}{Q} + C_c \frac{Q}{2} + PD$$

$$= \frac{(2{,}500)(200)}{(90)} + \frac{(190)(90)}{2} + (900)(200) = \$194{,}105$$

- Because $194,105 < $233,784, maximum discount price should be taken and 90 units ordered.

Winter 2016 ML&SCM 43

Quantity Discounts with Carrying Costs Percentage of Price Example (1 of 3)

- University Bookstore example, but a different optimal order size for each price discount.
- Optimal order size and total cost determined using basic EOQ model with no quantity discount.
- This cost then compared with various discount quantity order sizes to determine minimum cost order.
- This must be compared with EOQ-determined order size for specific discount price.
- Data:
  - C_o = $2,500
  - D = 200 computers per year

Winter 2016 ML&SCM 44

Quantity Discounts with Carrying Costs Percentage of Price Example (2 of 3)

Quantity	Price	Carrying Cost
0 - 49	$1,400	1,400(.15) = $210
50 - 89	1,100	1,100(.15) = 165
90 +	900	900(.15) = 135

- Compute optimum order size for purchase price without discount and C_c = $210:

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}} = \sqrt{\frac{2(2{,}500)(200)}{210}} = 69$$

- Compute new order size:

$$Q_{opt} = \sqrt{\frac{2(2{,}500)(200)}{165}} = 77.8$$

Winter 2016 ML&SCM 45

Quantity Discounts with Carrying Costs Percentage of Price Example (3 of 3)

Compute minimum total cost:

$$TC = \frac{C_o D}{Q} + C_c \frac{Q}{2} + PD = \frac{(2{,}500)(200)}{77.8} + 165\frac{(77.8)}{2} + (1{,}100)(200)$$

$$= \$232{,}845$$

Compare with cost, discount price of $900, order quantity of 90:

$$TC = \frac{(2{,}500)(200)}{90} + \frac{(135)(90)}{2} + (900)(200) = \$191{,}630$$

Optimal order size computed as follows:

$$Q_{opt} = \sqrt{\frac{2(2{,}500)(200)}{135}} = 86.1$$

Since this order size is less than 90 units , ***it is not feasible***, thus ***optimal order size is 90 units***.

Order Quantity for a Periodic Inventory System

- A ***periodic, or fixed-time period inventory system*** is one in which time between orders is constant and the order size varies.
- Vendors make periodic visits, and stock of inventory is counted.
- An order is placed, if necessary, to ***bring inventory level back up*** to some desired level.
- Inventory ***not monitored between visits***.
- At times, inventory can be exhausted prior to the visit, resulting in a stockout.
- Larger safety stocks are generally required for the periodic inventory system.

Order Quantity for Variable Demand

For normally distributed variable daily demand:

$$Q = \bar{d}(t_b + L) + Z\sigma_d\sqrt{t_b + L} - I$$

where:

$\bar{d}$ = **average demand rate**
t_b = **the fixed time between orders**
L = **lead time**
σ_d = **standard deviation of demand**
$Z\sigma_d\sqrt{t_b + L}$ = **safety stock**
I = **inventory in stock**

Order Quantity for Variable Demand Example

Corner Drug Store with periodic inventory system.

Order size to maintain 95% service level:

$\bar{d}$ = 6 bottles per day
σ_d = 1.2 bottles
t_b = 60 days
L = 5 days
I = 8 bottles
Z = 1.65 for 95% service level

$$Q = \bar{d}(t_b + L) + Z\sigma_d\sqrt{t_b + L} - I$$
$$= (6)(60 + 5) + (1.65)(1.2)\sqrt{60 + 5} - 8$$
$$= 398 \text{ bottles}$$

Winter 2016 ML&SCM 49

Example Problem Solution Electronic Village Store (1 of 3)

- For data below determine:
  - Optimal order quantity and total minimum inventory cost.
  - Assume shortage cost of $600 per unit per year, compute optimal order quantity and minimum inventory cost.
- Step 1 (part a): Determine the Optimal Order Quantity.

D=1,200 personal computers
C_c = \$170
C_o =\$450

$$Q = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2(450)(1{,}200)}{170}} = 79.7 \text{ personal computers}$$

Winter 2016 ML&SCM 50

Example Problem Solution Electronic Village Store (2 of 3)

$$\text{Total cost} = C_c\frac{Q}{2} + C_o\frac{D}{Q} = 170\left(\frac{79.7}{2}\right) = 450\left(\frac{1{,}200}{79.7}\right)$$
$$= \$13{,}549.91$$

Step 2 (part b): Compute the EOQ with Shortages.

Cs=\$600

$$Q = \sqrt{\frac{2C_oD}{C_c}\left(\frac{C_s + C_c}{C_s}\right)} = \sqrt{\frac{2(450)(1200)}{170}\left(\frac{600+170}{600}\right)}$$

=90.3 personal computers

Winter 2016 ML&SCM 51

Example Problem Solution
Electronic Village Store (3 of 3)

$$S = Q\left(\frac{C_c}{C_c + C_s}\right) = 90.3\left(\frac{170}{170+600}\right) = 19.9 \text{ personal computers}$$

$$\text{Total cost} = \frac{C_s S^2}{2Q} + C_c\frac{(Q-S)^2}{2Q} + \frac{C_o D}{Q}$$

$$= \frac{(600)(19.9)^2}{2(90.3)} + 170\frac{(90.3-19.9)^2}{2(90.3)} + 450\left(\frac{1{,}200}{90.3}\right)$$

$$= \$11{,}960.98$$

Example Problem Solution
Computer Products Store (1 of 2)

- Sells monitors with daily demand normally distributed with a mean of 1.6 monitors and standard deviation of 0.4 monitors. Lead time for delivery from supplier is 15 days.
- Determine the reorder point to achieve a 98% service level.
- Step 1: Identify parameters.

$$\bar{d} = 1.6 \text{ monitors per day}$$

$$L = 15 \text{ days}$$

$$\sigma_d = 0.4 \text{ monitors per day}$$

$$Z = 2.05 \text{ (for a 98\% service level)}$$

Example Problem Solution
Computer Products Store (2 of 2)

Step 2: Solve for R.

$$R = \bar{d}L + Z\sigma_d\sqrt{L} = (1.6)(15) + (2.05)(.04)\sqrt{15}$$

$$= 24 + 3.18 = 27.18 \text{ monitors}$$

Determining Reorder Point with Excel

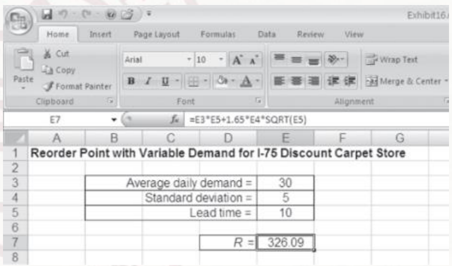


Exhibit 16.6

Winter 2016 ML&SCM 55

Order Quantity for the Fixed-Period Model Solution with Excel

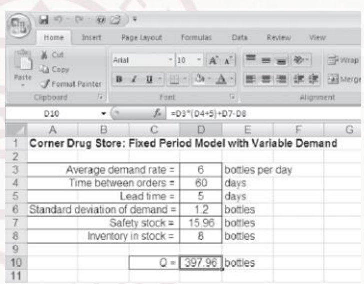


Exhibit 16.7

Winter 2016 ML&SCM 56